(1) **Ideal gas law**: balance of pressure (P), density (ρ), and temperature (T) for an ideal gas.

$$P = \rho RT, R = 287JK^{-1}kg^{-1} \tag{1}$$

- If T held constant, P is proportional to ρ
- If P held constant, T is inversely proportional to ρ [i.e., cold air is denser than warm air at the same P]
- Air rises if ρ of parcel is less than the environmental air [i.e., warm air rises]

(2) **First law of thermodynamics** is the conservation of energy If no heat is added to the system, the work done on the system plus the heat of the system are conserved:

$$\frac{C_p}{\Delta T} = \frac{\Delta P}{\rho}, Cp = 1004JK^{-1}kg^{-1} \tag{2}$$

From (1), we can replace P on the right hand side of (2) to eliminate ρ

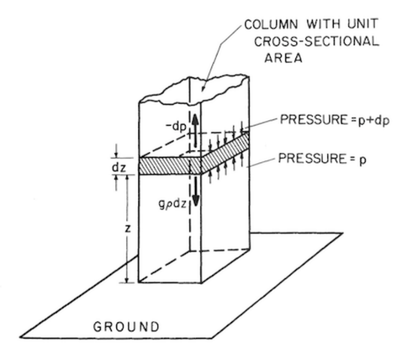
$$C_p \Delta T = -g\Delta Z \tag{3}$$

Solve for the the change in temperature with height for a system with no energy added/removed

$$\Delta T/\Delta Z = -g/C_p = 9.8K/1000m \tag{4}$$

- This is the dry adiabatic lapse rate, or the rate at which air cools when forced to rise w/o heat added/remove
- Latent heat release in moist environment reduces reduces the rate of temperature change with height
- The environmental lapse rate is typically below the dry adiabatic lapse rate, global mean is 6K/1000m
- C_p is the specific heat at a constant pressure

(3) **Hydrostatic balance**: vertical balance of gravitation and pressure gradient forces The figure to the right shows a thin slab of the atmosphere, with thickness of ΔZ. The gravitational force acts downward and is the mass of the slab ($\rho\Delta$ Z) times gravity. The pressure gradient force acts upward. Note that the pressure at the bottom of the layer (P2) ¿ than the pressure at the top of the layer (P1). Since pressure gradient force opposes the pressure gradient, this means the pressure gradient force is simply (P2-P1) or -ΔP.



$$-\Delta P = \rho g \Delta Z, or \Delta P/\Delta Z = -\rho g \tag{5}$$

- Change in pressure with height is proportional to the density of air in that column and inversely proportional to temperature
- The elevation above mean sea level of a given pressure surface is called the **Geopotential Height**

(4) **Hydrostatic balance**: vertical distance between two pressures as a function of temperature.

$$\Delta P/P = -g/RT\Delta Z \tag{6}$$

$$\Delta Z = R/g/\overline{T}log(P2/P1) \tag{7}$$

- The change in elevation between two pressure surfaces is proportional to the air temperature of that layer.
- This means that to go through half the mass of the atmosphere [from mean sea level at 1013.15mb to 500mb] requires a greater ascent in the equator than at the pole.
- We can refer to (7) as the **atmospheric thickness** between two layers